Invited Lecture

Mathematical Instruction and Textbook Use in Post-Secondary and Tertiary Contexts: A Discussion of Methods

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ABSTRACT In my work, I seek to understand how interactions between instructors, students, and resources — both inside and outside of the classroom, create opportunities for mathematics learning in post-secondary settings. Various methodological decisions have advanced this work. I showcase the evolution of two inter-dependent research strands that together have helped me understand the centrality of resource use by instructors and students and its implications for student learning.

Keywords: Instruction; Resources; Post-secondary education; Tertiary education.

I investigate how and why resources, instructors, and students interact to create rich opportunities for mathematics learning in post-secondary and tertiary settings. In this paper, aggregating findings from various studies, I reflect on the types of measures I have used to describe teachers' practice inside their classrooms and using textbooks. The presentation is organized in three sections. I present studies characterizing instruction first, followed by studies of textbook use. I conclude with a reflection on the methods used.

1. Characterizing Instruction

Following Cohen, Raudenbush, and Ball (2003) characterization of instruction as the interactions between instructor students and content allowed me to observe classroom activity occurring in real time. In earlier studies, using observations and audio-recordings of lessons, I relied on low inference codes, such as counts of audible speaking turns (i.e., speech that is given by a speaker before being interrupted or giving the floor to another speaker) or their length (in number of words) (Mesa, 2010a, 2010b). The counts of audible speaking turns can be identified by speaker, and a very simple ratio of number of student-turns to number of teacher- turns, when done at a large scale, provide important information about patterns of interaction between teachers and students. When the ratio is 1 then number of student- and teacher-turns are the same;

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ratios over 1 indicate more student-turns than teacher-turns. Using close 150 lessons from five different studies, ranging from developmental to graduate level courses in about 40 different institutions; in some studies, with maximum variation sampling (e.g., developmental to graduate courses) and homogeneous (e.g., community colleges, successful calculus institutions, inquiry-based learning) it is possible to find averages of these ratios, and they are quite revealing (Mesa, 2011). In Fig. 1, I present these ratios for various types of courses at the university and at community colleges[2] upper division and first year courses at university; pre-college and developmental level courses at community colleges, and inquiry-based learning courses at university. In university courses it is typical for teachers to dominate the talk in the class; in courses taught at community colleges, that is not the case, and in courses that use inquiry, the difference is more remarkable, at least relative to other courses taught at university (Mesa, 2009, 2011).

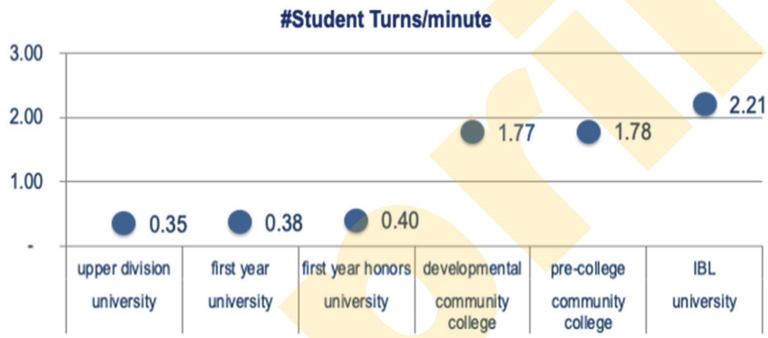

Fig. 1. Ratios of student turns to teacher turns by type of course and setting

Counts of words, is also revealing; a teacher-turn on average is 40 words, whereas student turns are on average between four and five words. Student turns that are between one and three words can be about 51% in non-inquiry classes, but less than 10% in inquiry classes. This suggests that even when students participate in classroom, their contributions are limited, except when the classes use inquiry. These results seem to make sense when we think that during mathematics classrooms, the prevalent mode of instruction is lecturing (Mesa and White, 2022), whereas in inquiry-based classrooms, students then to either speak among themselves and ask each other questions, or present information at the board without intervention from the teachers. Thus, these low inference codes corroborate classroom participation patterns that we know exist in post-secondary classrooms.

These codes though are insufficient to further characterize the quality of the interactions. To look at these, I have relied on analyses of examples and questions that teachers use in classroom under the assumption that the content and ways in which these are phrased have the potential of triggering a particular cognitive process in students. To analyze examples and tasks, I initially used the revised Bloom's taxonomy

[2] Community colleges are post-secondary institutions that prepare students for professional vocational work and that provide courses equivalent to the first two years of a college degree.

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(Anderson et al., 2001), which classifies knowledge into four distinct dimensions — factual, procedural, conceptual, and metacognitive — and identifies six different types of cognitive processes — each increasing in complexity, as they are assumed to require more cognitive resources: Remember, Understand, Apply, Analyze, Evaluate, and Create Fig. 2). Using this taxonomy with small modifications, it has been possible to classify tasks used in classrooms (Mesa, 2010b) and in homework and in exams (White and Mesa, 2014). Classifying questions into non-mathematical and mathematical (Novel or Routine) has also been useful to gauge the cognitive demand of mathematical work done in classrooms (Mali et al., 2019, Meta et al., 2014). Naturally, inferring the cognitive demands of examples used and questions asked is more difficult, as it requires rigorous training and understanding of the context in which the questions and problems are asked: A question or a problem asked in a calculus class might be of low cognitive demand because the students might already be familiar with the content, whereas the same question or problem asked in college algebra class might be novel. Thus, all these analyses are paired up with the context in which the courses take place, including the place in the sequence of the course, the course objectives, the profiles of the students taking the class, and the goal of the course in a student's major.

Knowledge Dimension	*Cognitive Processes Dimension*
1. **Factual Knowledge:** Basic elements students must know to be acquainted with a discipline or solve problems in it, including knowledge of terminology and of specific details. 2. **Conceptual Knowledge:** Interrelationships among the basic elements within a larger structure that enable them to function together. It involves knowledge of classifications and categories, of principles and generalizations, and of theories, models, and structures. 3. **Procedural Knowledge:** How to do something, method of inquiry, and criteria for using skills, algorithms, techniques, and methods. It includes knowledge of subject-specific skills and algorithms, of specific techniques and methods, and of criteria for determining when to use appropriate procedures. 4. **Metacognitive Knowledge:** Knowledge of cognition in general as well as awareness of one's own cognition. It includes strategic knowledge, knowledge about cognitive tasks (including appropriate contextual and conditional knowledge), and self-knowledge.	1. **Remember:** Retrieve relevant knowledge from long-term memory, including recognizing and recalling. 2. **Understand:** Construct meaning from instructional messages, including oral, written, and graphic communication. It involves interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. 3. **Apply:** Use a procedure in a given situation. It involves executing and implementing. 4. **Analyze:** Break material into its constituent parts and determine how the parts relate to one another and to an overall structure or purpose. It involves differentiating, organizing, and attributing. 5. **Evaluate:** Make judgments based on criteria and standards. It involves checking and critiquing. 6. **Create:** Put elements together to form a coherent or functional whole and reorganize elements into a new pattern or structure. It involves hypothesizing, designing, and producing

Fig. 2. Revised Bloom's Taxonomy (Anderson et al., 2001)

These analyses, using a similar corpus of data, have revealed different proportions of routine tasks and questions asked in classes in which the mode of instruction is lecturing, versus classes in which the mode of instruction is inquiry as seen in Fig. 3.

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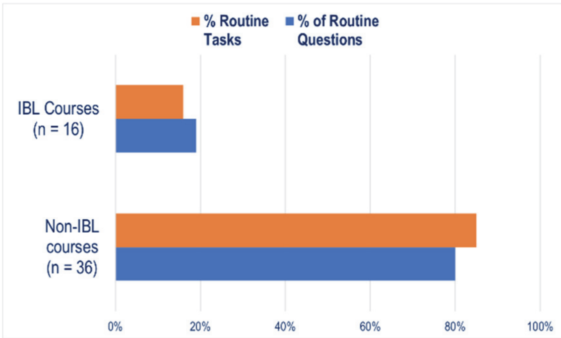

Fig. 3. Proportion of routine tasks and questions by mode of instruction.

We see a stark contrast in the cognitive demand for the cognitive demand of questions and problems in courses that are inquiry versus those that are not inquiry. In 16 inquiry courses, the proportion of routine questions or tasks is under 20 percent, whereas in other courses, it is above 80 percent. One can infer that it is likely that the cognitive demand expectations of work assigned to students in the two types of courses is different. These differences might lead us to think twice about the advantages of continuing using non-inquiry lessons in our mathematics courses.

To further investigate this phenomenon, and as part of a national study of calculus in the United States (Bressoud, 2013; Bressoud et al., 2013; Bressoud et al., 2015; Bressoud and Rasmussen, 2015), we analyzed the problems solved during calculus lessons attending to four aspects of the reform: student involvement, use of representations, features of the problems, and technology. Coding problems as having more or less of each of these features, allowed us to create a sort of heatmap for each lesson observed, that identified for each of the four aspects the extent to which reform was enacted (see Fig. 4).

Student Involvement (0-3)	Representation Use (0-2)	Feature Use (0-2)	Technology Use (0-1)
0: Instructor only	0: Symbolic only	0: Only practicing skills or known methods	0: No technology
1: Individual work and Instructor only	1: One representation that is not symbolic	1: Context and Diagram, but no Proof/Justification, Multiple methods, Open Ended	1: Some technology
2: Group, pair, class discussion, no student presentations	2: Any two representations	2: Proof/Justification, Multiple methods, Open Ended	
3: Student presentations			

Fig. 4. Categorization of reform features in calculus lessons (Mesa and White, 2022; White and Mesa, 2018).

The gray color represents "business as usual" whereas darker shades of yellow more enactment of reform features. We can compare now, full lessons in terms of the level of reform enacted. These two lessons from the same institution, taught by two

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different instructors show important differences, with the lesson on the left (Fig. 5a) enacting more elements of reform than the lesson on the right (Fig. 5b) — which appears mostly gray. These representations do suggest that students in these two courses, taught at the same institution, are experiencing mathematics differently.

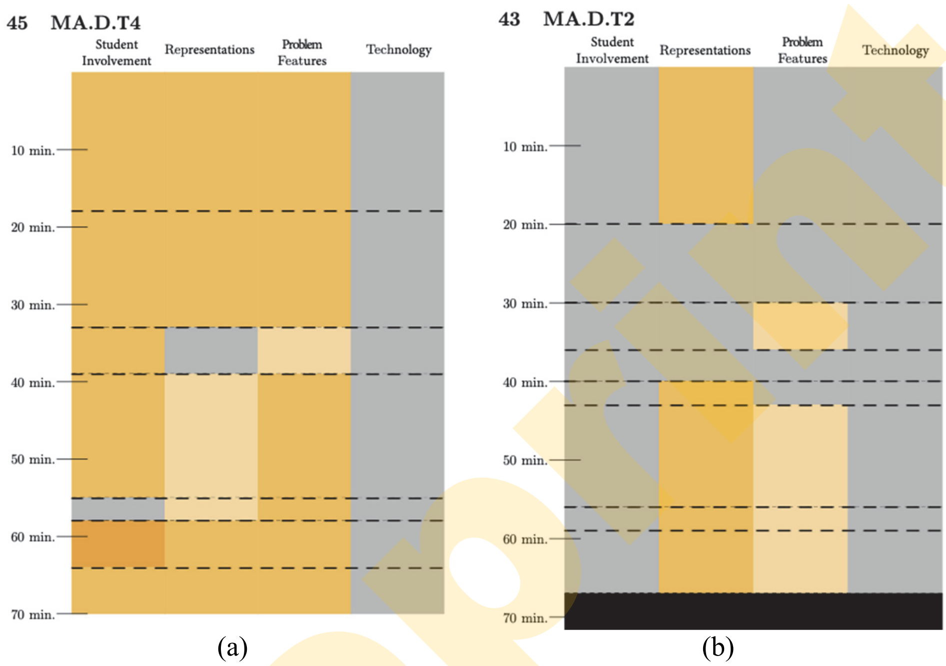

Fig. 5. Maps of two calculus lessons taught by two different instructors at the same institution.(Mesa and White, 2022)

Now, while analyzing tasks and problems is useful, and as seen here, instruction is more complex than that. Understanding what hoes into high quality instruction involves not only the types of problems and questions that teachers select; these decisions attend merely to the teacher content interaction of the definition of instruction and only superficially address the student content interaction. To understand how instruction operates, a more complex system of observation is needed.

Building up on work done with elementary school practitioners (Heather et al., 2008; Heather C. Hill, Blunk, et al., 2008; Heather et al., 2005; Heather et al., 2004), we have developed an instrument that documents the quality of instruction in algebra courses taught at community colleges; the instrument, *Evaluating the Quality of Instruction in Postsecondary Mathematics,* EQIPM, addresses the interactions between students, teacher, and content with 14 codes, grouped into three distinct hypothesized dimensions, as seen in Fig. 6[3]. The 14th code is hypothesized to relate to all the dimensions. A categorical confirmatory analysis revealed that these are distinct

[3] The definition of the dimensions and the codes is provided in the Appendix.

dimensions (Lamm et al., accepted). The final instrument includes 12 codes (Tab. 1)

Evaluating the Quality of Instruction in Postsecondary Mathematics (EQIPM)		
Quality of Student-Content Interaction	Quality of Instructor-Content Interaction	Quality of Instructor-Student Interaction
1. Student Mathematical Reasoning and Sense Making 2. Connecting Across Representations-Student 3. Situating the Mathematics-Student	4. Instructors Making Sense of Procedures 5. Connecting Across Representations-Instructor 6. Situating the Mathematics-Instructor 7. Supporting Procedural Flexibility 8. Organization in the Presentation of Procedures 9. Mathematical Explanations	10. Instructor-Student Continuum of Instruction 11. Classroom Environment 12. Inquiry/Exploration 13. Remediation of Student Errors and Imprecisions
14. Mathematical Errors and Imprecisions in Content and Language		

Fig. 6. Codes for the EQIPM instrument (Mesa, Duranczyk, Watkins, & AI@CC Research Group, 2019, February; Mesa et al., 2020)

Tab. 1. Three-factor ordinal confirmatory factor analysis results with *Remediation of Student Errors and Difficulties* removed.

Factor	Item	Std. Loading	SE
Student-Content Interaction	Student Mathematical Reasoning and Sense Making	0.778	0.058
	Student Connecting across Representations	0.722	0.061
	Student Situating the Mathematics	0.531	0.127
Instructor-Content Interaction	Instructor Making Sense of Mathematics	0.629	0.078
	Instructor Connecting across Representations	0.463	0.083
	Instructor Situating the Mathematics	0.324	0.091
	Mathematical Explanations	0.456	0.101
	Supporting Procedural Flexibility	0.403	0.104
	Organization in the Presentation	0.666	0.097
Instructor-Student Interaction	Instructor-Student Continuum of Instruction	0.890	0.081
	Classroom Environment	0.788	0.110
	Inquiry / Exploration	0.599	0.095

Chi-Square = 58.004, p value 0.177; CFI = 0.967; RMSEA = 0.039; SRMR = 0.074

These findings across multiple studies using different types of analytical tools that demand low and high inference coding, strongly suggest that typical mathematical instruction in post-secondary and tertiary levels is constituted by interactions primarily led by the instructor, with limited student participation, on tasks that are mostly routine and with low reform activities (such as collaborative work, student presentations, multiple representations used and significant use of technology), and that mathematics lessons in which inquiry is used do not follow these patterns.

With an instrument the available instrument, it would be possible to seek to identify cases in which instruction is of high quality, possibly in inquiry-based contexts, or professional development programs based on the items in the instrument; the instrument could be complemented with other instruments that assess inclusion,

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diversity, and equitable practices.

2. Studying Textbook Use

Research on mathematics textbook use by faculty and students is in its infancy. There are over a dozen of studies about undergraduate students' use of mathematics textbooks, and a handful of studies about how mathematics faculty use their textbooks for teaching. A main reason for the scarcity of research is methodological, as it is difficult to conduct such work in naturalistic settings, that is, in real classrooms with real faculty and their students. Most of the existing work has been conducted in laboratory like settings, with a handful of participants, and have focused on how people understand or make sense of what they read (Shepherd et al., 2010; Sierpinska, 1997; Weinberg and Wiesner, 2011; Wiesner et al., 2020). Some studies have investigated student use via surveys (Gueudet and Pepin, 2018; Weinberg et al., 2012). How can textbook use be studied in actual classrooms? The problem becomes more complex when studying online textbooks. What methodological tools are there to study textbook use?

Textbooks are an integral part of teaching and learning activities in post-secondary mathematics; not only are they a source for inspiration for organizing presentations and activities for classroom work, but they are also fundamental for designing homework or other student assessments. As part of a large scale study digital textbook use (Beezer et al., 2018), we selected three textbooks, Active Calculus[4] (herafter AC, Boelkins, 2021), A First Course in Linear Algebra[5] (hereafter FCLA, Beezer, 2021), and Abstract Algebra Theory and Applications[6] (hereafter AATA, Judson, 2021), chosen because they target different profiles of students in a mathematics program. These textbooks are written in PreTeXt[7] a markup language that provides a structure to the content of the textbooks, and a unique identifier for each textbook element. The textbooks can include computation cells (in Sage[8]) and are highly linked. They can also be rendered in any device without losing the quality of the mathematics or the graphs and can be printed in any format, including Braille.

The textbooks used in the project, have been modified to facilitate interactivity: in response to specific questions, students can type answers, and when they submit them, teachers can read their answers in real time. If students had submitted answers to the questions as preparation for the class, teachers could in a quick glance, make decisions about what to address in the lesson, perhaps change and example, or perhaps skip one that might not be needed. The blank spaces appear after each question in the Preview Activities in the calculus textbook, and the Reading Questions, in the other two

[4] https://books.aimath.org/ac/frontmatter.html
[5] https://books.aimath.org/fcla/front-matter.html
[6] https://books.aimath.org/aata/frontmatter.html
[7] https://utmost.aimath.org/pretext/
[8] https://www.sagemath.org/

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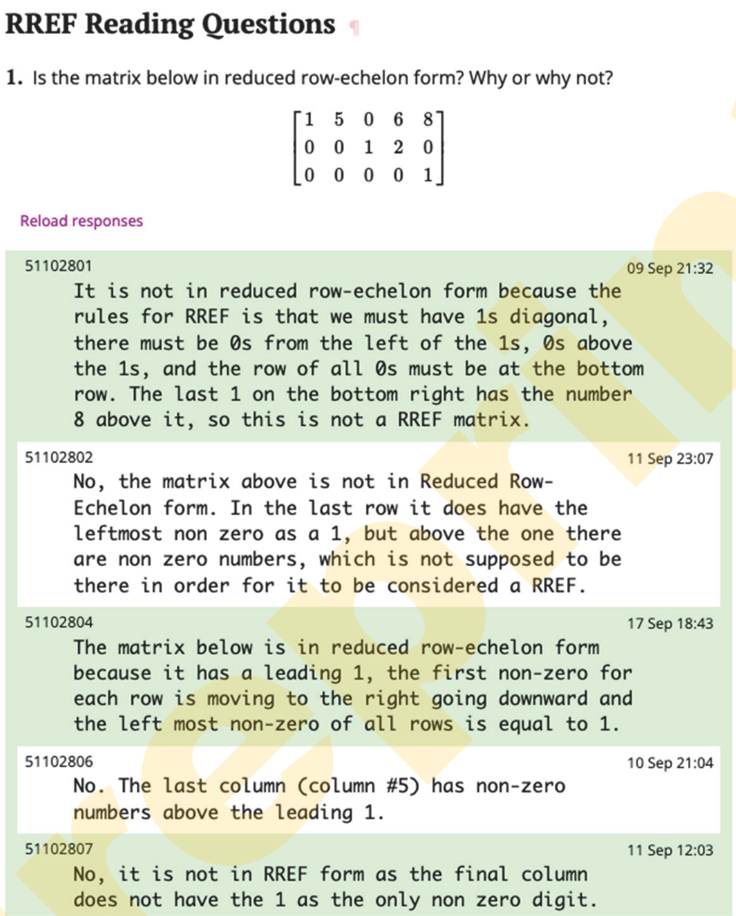
textbooks. The Reading Questions for AATA appear at the end of each chapter, whereas in the other textbooks, the feature appears within each section of the chapter (see Fig. 7).



RREF Reading Questions

1. Is the matrix below in reduced row-echelon form? Why or why not?

$$\begin{bmatrix} 1 & 5 & 0 & 6 & 8 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Reload responses

51102801 09 Sep 21:32
It is not in reduced row-echelon form because the rules for RREF is that we must have 1s diagonal, there must be 0s from the left of the 1s, 0s above the 1s, and the row of all 0s must be at the bottom row. The last 1 on the bottom right has the number 8 above it, so this is not a RREF matrix.

51102802 11 Sep 23:07
No, the matrix above is not in Reduced Row-Echelon form. In the last row it does have the leftmost non zero as a 1, but above the one there are non zero numbers, which is not supposed to be there in order for it to be considered a RREF.

51102804 17 Sep 18:43
The matrix below is in reduced row-echelon form because it has a leading 1, the first non-zero for each row is moving to the right going downward and the left most non-zero of all rows is equal to 1.

51102806 10 Sep 21:04
No. The last column (column #5) has non-zero numbers above the leading 1.

51102807 11 Sep 12:03
No, it is not in RREF form as the final column does not have the 1 as the only non zero digit.

Fig. 7. Teacher view of responses provided by students to one of the reading questions in the FCLA course, in the section Reduced Row Echelon Form (https://books.aimath.org/fcla/section-RREF.html)

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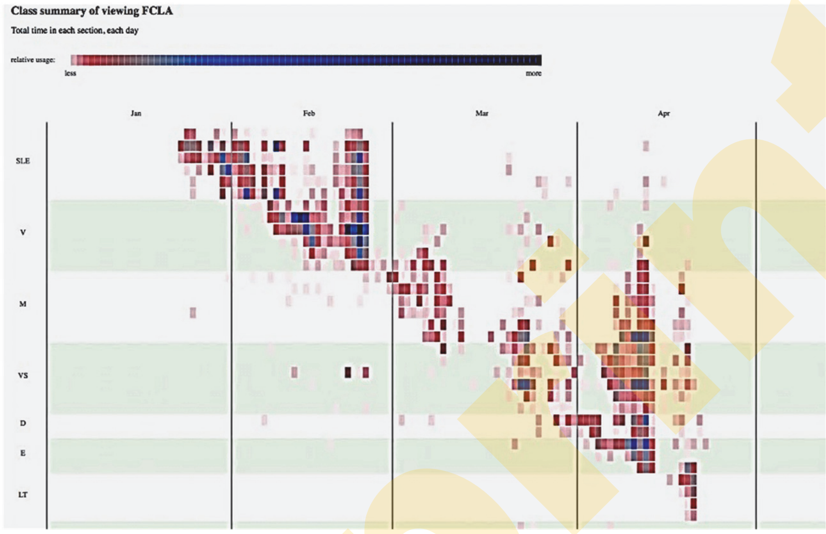
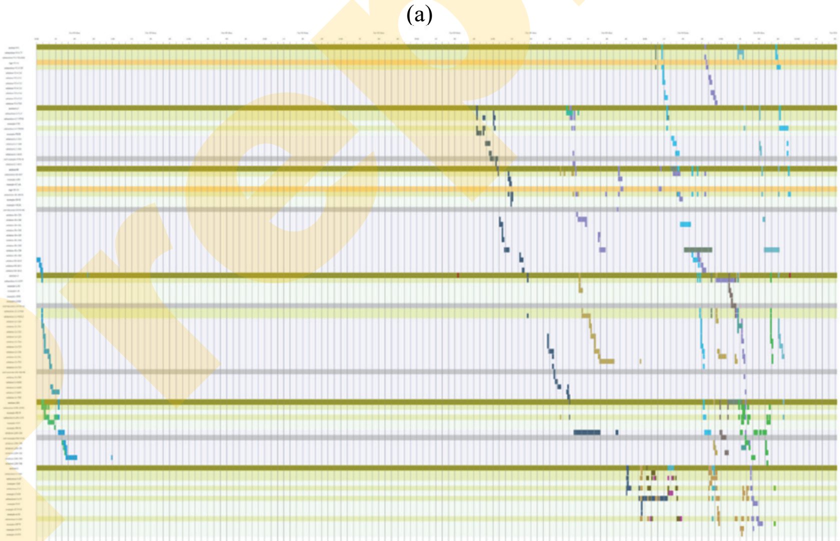
To interact with the feature, the users need a username and a password, which facilitates the identification of how much time a user spent viewing a particular section of the textbooks in any given day. This information is used to create heatmaps of use, heatmaps that can be created at various levels of detail: course, chapter, section, and user (see Fig. 8).

For this project we collected data from over 55 instructors and their students (close to 900), over ten semesters; we collected a wide array of data: surveys, tests of knowledge, lecture notes, periodic surveys with open and close ended questions, and for a few sections, an intensive data collection process including observations, interviews, and student focus groups. Fig. 9 illustrates the process of data collection within any given semester.



(a)



(b)

Fig. 8. Heatmap viewing data
(a) at the course level; (b) at the user level. different colors reflect different users.

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	Pre term	Week in the term 3	6	9	12	15	Post term	Summer
Teacher surveys	X							
Teacher logs		X	X	X	X	X		
Course syllabi		X						
Lecture notes[a]				X				
Workshop								X
Computer-generated student and teacher viewing data[b]								
Campus visits[c]				X				
Student logs		X	X	X	X	X		
Student surveys				X				
Students tests		X				X		
Student grades							X	

Notes: [a]. Varies by textbook. [b]: Collected continuously throughout the term. [c]: Includes classroom observations, teacher interviews, and a student focus group.

Fig. 9. Data collection for the UTMOST project.

We found that students report extensive use of their textbooks, in particular features such as definitions, examples, and theorems, as they prepare their homework or study for exams; they also make extensive use of the narrative text and of the preview activities or reading questions as they prepare for class. We identified expected differences by textbook. AC students read the narrative text, focus on activities, and complete the WeBWorK practice problems; FCLA and AATA users, attend to definitions, theorems, and proofs. Sage use varies substantially by instructor. It is remarkably difficult to establish any connection between time spent viewing the textbook ad how well instructors think their students are doing in the course. Many faculty reported being surprised that students who appeared to be viewing the textbook a lot may do well, not well, or just OK in the course. The same occurred for students who did not view the textbook much. Part of the reason has to do with the number and variety of resources students report using, including other textbooks, online forums or video sites, peers, family, tutoring, and their instructors. Some students preferred using a printed copy of the textbook and thus their viewing did not get recorded. An internal quantitative analysis confirmed prior research that indicates that students who are more motivated perform better in their courses (earn higher grades in the course). Having a textbook available online or in printed form does not seem to be of any advantage measurable by grades or gains in knowledge.

The multiple data sets collected as part of this project and the sheer number of participants has led us to data reduction and analysis techniques that use natural language processing (Kumar et al., 2016) and networks of knowledge graphs (Hamilton, 2017). Using both manual coding (Mesa and Mali, 2020) and natural language processing, we have identified multiple uses students give to their textbooks, as anticipated by the instrumentational approach (Rabardel, 2002), our theoretical framework, which defines instruments as a combination of artifacts and schemes of use; the schemes of use encompass goals, rules of actions, operational invariants, and

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possibilities for inferences (Vergnaud, 1998). We have been able to identify that students read the textbooks to reverse engineer processes shown in examples, to check their work, to learn definitions, to understand how proofs work, to anticipate what will happen in class, to propose questions that might be useful for their learning, and for self-directed study. Faculty tend to think that students in lower division courses only read the textbook to find out examples that can help with their homework (Mesa & Griffiths, 2012). This is not the case. Students will read the textbook is asked by their instructors and will use it for their own learning. Instructors too, use textbook features, such as the reading questions to identify whether they have understood the material that they are about to teach (Mesa et al., 2021). Moreover, the data collected from viewing the textbooks can be reliably mapped to the uses students describe for particular features (Kanwar and Mesa, accepted). Instructors, in using their textbooks for planning, also use a wide range of resources, from graduate school notes, or their own notes from prior terms, or the internet, or repositories available in their campus. Instructors may attend to the course objectives or to student thinking as they plan or to a combination of both (Liakos et al., in press; Mesa, accepted). We have confirmed that instructors will integrate the textbook into their usual ways of planning and enacting instruction — even though the textbooks are open source, making changes to those textbooks is very difficult at this time. We have also learned that students use their textbooks in more ways than other studies report, in part thanks to the continued follow-up and because of the use of the heatmaps that assist them in recalling how they are viewing their textbooks; we have also made their teachers aware of the many ways the students use their textbooks, thus amplifying their knowledge of student work with resources. Finally, we have demonstrated that making connections between amount of textbook viewing and student performance is problematic because the reasons why users view and use textbooks and resources vary and because in general, they take advantage of more resources beyond their own textbooks. There is simply no way to connect time spent on a single resource and performance, as measured as a grade in a course.

3. Concluding Thoughts

My work to understand instruction and the use of textbooks in teaching and learning has required a wide range of methodological approaches that allow making coarse descriptions of practice (when using low inference codes) to descriptions that are more nuanced but that require much interpretation. The accumulation of data across terms and studies have increased the potential for pattern identification that has led to significant generalizations. The current developments in data science and artificial intelligence can be seen as great tools that can harness the potential of analyzing large bodies of data with higher levels of accuracy and reliability. The availability of these tools in online environments make it possible the data collection of participants who do not need to be closely located to a researcher. If we have learned something about the pandemic of the COVID-19 (2019–2022) is that we are more connected than ever,

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and that proximity is not a prerequisite for having meaningful interchanges about work.

There are also ethical questions about the handling of the vast information that is being gathered. While we still rely on institutional review boards to ensure that the use of personal information is done correctly in ways that protect the users, we still rely on mechanisms put in place before the advent of these new technologies. Ensuring that such information be managed in ways that will not harm participants is a key point to attend to. An important question remains open: What is the potential of the interactions between resources, teachers, the students, and the content to create rich opportunities for mathematics learning in post-secondary and tertiary settings? Much work needs to be done, but my sense is that most work is methodological is we are interested in learning about resource use in naturalistic settings.

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Appendix: EQIPM Dimensions (in gray boxes) and Items (in white boxes).

Dimensions of Quality
Student-Content Interaction: Attends to actions that demonstrate students' thinking about the mathematics by either describing their own thinking and reasoning, making connections among representations of mathematical ideas, or stating relationships with other mathematical topics and ideas they had seen before
Student Mathematical Reasoning and Sense-Making: Assesses student utterances that showcase reasoning and sense-making about mathematical ideas.
Connecting Across Representations-Student: Assesses connections that students express within, between, and across representations of the same mathematical problems, ideas, and concepts.
Situating the Mathematics-Student: Assesses connections students express to other aspects of the algebra curriculum, related topics, or the broader domain of mathematics, situating and motivating the current area under study within a broader context.
Instructor-Content Interaction: Attends to actions that reflect instructors' engagement with the content by making sense of the mathematics, making connections among representations of mathematical ideas explicit, situating the content in the larger structure of mathematics, describing the procedures not just as steps to follow but attending to why the steps are required or what other procedures can do the same job, presenting the ideas in a coherent organization, and providing

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mathematically sound explanations
Instructors Making Sense of Mathematics: Assesses how instructors leverage known and new mathematical ideas or students' personal knowledge or experiences, in order to make meaning of the mathematics presented.
Connecting Across Representations-Instructor: Assesses connections that instructors express within, between, and across representations of the same mathematical problems, ideas, and concepts.
Situating the Mathematics-Instructor: Assesses connections instructors express to other aspects of the algebra curriculum, related topics, or the broader domain of mathematics, situating and motivating the current area under study within a broader context.
Mathematical Explanations: Assesses how instructors provide mathematical reasons and justification for why something is done.
Supporting Procedural Flexibility: Assesses how instructors support the development of procedure use by identifying what procedure to apply and when and where to apply it.
Organization in the Presentation: Assesses how complete, detailed, and organized the instructor's presentation of content is when outlining or describing the mathematics or describing the steps used in a procedure.
Instructor-Student Interaction: Attends to actions describing how the instructor and the students relate to each other, specifically how the instructor shares the instructional space with the student, how the class environment supports students' participation and learning, how opportunities are created for students to engage in mathematical exploration, and how errors and difficulties are managed.
Instructor-Student Continuum of Instruction: Assesses the degree to which either the instructor or the students contribute to the development of the mathematical ideas (abstract concepts, formulas, notation, definitions, concrete examples, pictorial examples, and rules/properties). It captures who is responsible for the development of those ideas.
Inquiry / Exploration: Assesses the degree to which mathematics exploration and inquiry occurs.
Classroom Environment: Assesses how instructor and students create a respectful and open environment in their classroom in which expectations for high quality mathematical work is the norm.
Remediation of Student Errors and Difficulties: Assesses remediation (either for the whole class or with individuals/small groups) in which student misconceptions and difficulties with the content are addressed by attending to their reasoning.
Cross-Cutting Item: Mathematical Errors and Imprecisions in Content or Language: Assesses mathematically incorrect or problematic use of mathematical ideas, language, or notation by students and instructors. This item is scored differently than the rest of the items in the instrument, with a rating of 0 indicating no errors or imprecisions present, and a number between 1 and 4 to indicate low to high severity of the errors and imprecisions.

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